

Calculation of Minimum Variance Hedge Ratios

Portfolio Variance

The formulas in this note are loosely based on the descriptions given in Clark and Kritzman (AIMR: *Currency Management: Concepts and Practices*, 1996), although I have modified their notation to be consistent with other documents published by Alford Associates and papers I have written. The equations given here illustrate only the case of a single domestic investment coupled with a single foreign investment. This combination results in a need for three variance terms (one for each asset and one for the currency), three covariances, and a measure of net currency exposure (i.e. the proportion of currency exposure left unhedged). The math gets a little more complicated if more than one foreign currency is involved, but the concepts illustrated here remain valid. In real life, there are many different approaches to this idealized analysis, recognizing the trade-off between mathematical elegance and real-world instabilities.

Currency Exposure

In the formulas below, “E” denotes the (net) amount (on a scale of 0 to 1) of foreign currency exposure. If no hedging is done, this variable will be equal to the weight (**w**) of the foreign investment. If the position is fully hedged, E will be equal to zero, and the portfolio returns will have no sensitivity to changes in the exchange rate. If you prefer to think in terms of the proportion hedged (the hedge ratio, **h**):

$$h = E - 1$$

Variance of a Portfolio Containing

One Domestic (d) and One Foreign (f) Asset

$$\begin{aligned} \sigma_R^2 = & w_d^2 \sigma_d^2 + w_f^2 \sigma_f^2 + 2w_d w_f \text{Cov}_{df} \\ & + E^2 \sigma_c^2 + 2E(w_d \text{Cov}_{dc} + w_f \text{Cov}_{fc}) \end{aligned}$$

Minimum Variance Currency Exposure

$$E_{\min} = - \frac{w_d \text{Cov}_{dc} + w_f \text{Cov}_{fc}}{\sigma_c^2} = - (w_d \beta_d + w_f \beta_f)$$

Thus, **h** ranges from 0 to -1. In practice, of course, these variables (**h** and **E**) are not limited the ranges I have stated, since it is possible to *overhedge* (i.e. have a net short exposure to a currency) or to own a long currency forward in a currency for which there is no underlying asset.

As the first equation on the previous page illustrates, when considering foreign assets (i.e. investments denominated in a currency other than one's own), it is necessary to take into account the effect of not just the covariance of the returns of the domestic asset (**d**) with those of the foreign asset (**f**), but also the covariance of the currency exchange rate (**c**) between the two countries with each of the assets in question. **NB:** the foreign asset returns are calculated in local (i.e. foreign) currency terms.

Tracking Error

Note that it is a fairly straightforward matter to use this equation to calculate tracking error instead of total variance. In order to do that, it is necessary only to substitute into the equation the differences of portfolio weights from the benchmark weights.

Minimum Variance Hedge Ratios

The second equation gives the calculation for the minimum variance exposure (hedge ratio) with respect to a similar two-asset portfolio. Here, too, the equation could be translated into a minimum tracking error currency exposure *difference* by substituting weight differences into the equation.

Note that the betas given in the equation refer to the sensitivity of each asset with respect to the currency returns. Thus, if more than one currency is involved, the sensitivity of each asset with each currency must be calculated. Fortunately, for practical purposes, this turns into a fairly straightforward regression exercise. Based on a reasonable time period (say, five years of monthly data), the historical beta of each asset with the currency in question can be calculated, and the minimum variance hedge ratio (**h**) or currency exposure (**H**) can be easily computed by simply taking a weighted sum of all the betas.

One final note of clarification: the **c** terms all refer to the variance of changes in the currency exchange rate when viewed from the perspective of the domestic currency.

Practical Application

Active managers who deviate from their benchmark weights with respect to a given currency often wonder if they should neutralize this difference in the absence of an opinion about the currency. Global portfolio managers should be keenly interested in the tracking error version of the minimum variance hedge ratio. This number will give the proper amount of currency hedging to be done to minimize risk from currency exposure in the absence of an opinion about the direction of the spot rate. The answer will never be “all” or “none”.

This report was prepared by
Michael F. Wilcox, CFA

michael.wilcox@alfordinc.com

Additional information is available upon request.